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LONG-RANGE REENTRY GUIDANCE OF A LOW L/D VEHICLE

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SUMMARY

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The guidance scheme analyzed for the atmosphere entry phase ... mission is based upon the linear theory of perturbations about a nominal or reference trajectory. The scheme differs from the classical linear theory primarily through the use of empirically determined weighting factors. The guidance shceme uses a single nominal trajectory to make it simple and easy to store for use by the on-board computer.

It is shown that this guidance scheme, when applied to a typical $(L/D)_{max} = 0.4$, roll modulated vehicle, could provide accurate guidance over ranges from 1,500 to 12,000 statute miles for entry angles which include virtually all of the vehicle's capability.

Results are also presented for the effects on guidance capability of certain off-design conditions; namely, the effects of reentries from selected abort conditions, the effects of variations in lift-drag ratio, and the effects of atmospheric density deviations.

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INTRODUCTION

The guidance of aerospace vehicles requires information regarding the future consequences on the vehicle trajectory of a given control action. This information is predictable through the solution of the equations of state (equations of motion). Schemes which have been proposed for aerospace vehicle guidance can be loosely classified according to the manner in which these solutions are obtained. Most schemes fall into one of three broad categories, namely:

(1) Exact numerical solutions of the nonlinear equations of state.

(2) Approximate closed-form analytical solutions of the equations of state.

(3) Approximate numerical solutions in the neighborhood of an exact precomputed numerical solution (nominal trajectory) of the equations of state.

Examples of guidance schemes representative of the first two categories are given in references 1 and 2, respectively. The present report will be concerned with the guidance concept corresponding to category (3). This concept of guidance about a nominal trajectory has received considerable attention. It has been investigated for use both in the midcourse phase of the lunar mission

Uncand for the atmosphere reentry phase (e.g., refs. 3 to 6). The restriction to the neighborhood of the nominal trajectory forms the primary limitations on the range of applicability; this is particularly restrictive in the case of atmosphere entry.

The various proposed schemes employing this concept for atmosphere reentry have, in general, demonstrated an ability to guide to ranges of approximately 6,000 miles or less. Theoretically, the proposal of reference 6 to store multiple nominal trajectories and associated feedback gains should provide accurate guidance to any range, limited only by the storage capacity of the guidance computer. The on-board computer is subject to limitations on size, weight, and power requirements. These restrictions usually result in limitations on computer speed and information storage capacity, which, in turn, place a premium on guidance scheme simplicity.

The purpose of the present investigation was to determine if the guidance of a lunar mission type capsule to ranges of the order of one-half the earth's circumference could be accomplished by means of the nominal trajectory guidance concept. In the interests of simplicity, a single nominal trajectory was to be used. The results presented show that with relatively small modifications to the classical linear theory solution, it is possible to meet these objectives.

NOTATION

A aerodynamic acceleration, g units

F_{α}^{η} linear theory gain for the α state variable used to determine the magnitude of the control variable η, dimensions of $\frac{\eta}{\alpha}$ (See appendix A.)

g surface gravity, ft/sec^2

h altitude, ft

K_A, $K_{\dot{h}}$ empirical dimensionless weighting factors

$\frac{L}{D}$ lift-drag ratio, dimensionless

m mass, lb-sec^2/ft

r $r_e + h$, ft

r_e earth radius, ft

S vehicle reference area

[redacted]

t time, sec

u external force variable (See appendix A.)

V	total velocity
x	downrange
y	crossrange
x_{TG}	range to go, $x_f - x$
X,Y,Z	inertial axis system positive north, east, and radially outward (See fig. 10.)
$(\dot{\ })$	derivative with respect to independent variable
γ	flight-path angle (see fig. 10), deg
$\delta(\ )$	difference between actual and reference value of any quantity, $(\ ) - (\ )_r$
λ_α^η	$\left(\frac{\partial\eta}{\partial\alpha}\right)$ adjoint variable
Λ	angle of latitude, deg
ζ	heading angle (see fig. 10), deg
ψ	angle of longitude, deg
μ	product of universal gravitational constant and mass of planet, ft^3/sec^2
β	atmospheric density decay parameter, 1/ft
ρ	atmospheric density, $lb\text{-}sec^2/ft^4$

Subscripts

f	final
i	initial
h	horizontal component
m,p	summing index
r	reference or nominal
v	vertical component

DEVELOPMENT OF GUIDANCE SYSTEM CONTROL EQUATION

Basic Control Equation

The basic form of the control equation determined by linear perturbation theory is given in appendix A by equation (A11) as;

$$u_p(v) = u_{p_r}(v) + \sum_M F_{x_m}^{u_p}(v)\delta x_m(v) \tag{1}$$

Equation (1) is applicable to the control of any number of quantities in three dimensions, restricted only by the simplifying assumption made in appendix A that the number of control variables is equal to the number of quantities to be controlled. The following specific guidance system is developed on the basis of a two-dimensional (altitude, downrange) analysis for a typical lunar mission type capsule. The quantity to be controlled is the final range. It is shown in appendix B that, as a result of the limited crossrange capability of this type of vehicle, the results obtained are valid for application in three dimensions.

As formulated in appendix A, equation (1) defines a specific mode of control but does not define what dependent and independent variables are to be used. The choice of variables and the modifications made to equation (1) are discussed below.

Choice of Variables

Insofar as the theory is concerned, the particular set of state variables (altitude, range, etc.) and the independent variable (time or a state variable) chosen is completely arbitrary. There are practical reasons for choosing a state variable rather than time as the independent variable in order to reduce M in equation (1) by one, thus simplifying the guidance equations and reducing the computer storage requirements. In the present investigation, total velocity was chosen as the independent variable. This choice is not new (see, e.g., refs. 1, 2, and 4) and was made in the present investigation because of the simplification just noted, and because it appears that it has certain additional advantages. One of the advantages may be visualized by considering the consequences of using time as independent variable. The total time required to complete an atmosphere reentry depends almost directly on the total range traversed. Now the actual state variable values and those on the reference trajectory are compared (the δx_m of eq. (1)) at each value of the independent variable. Therefore, in order that a reference trajectory variable be available for guidance purposes, no trajectory which requires more time to complete than the reference trajectory may be considered. In practice this requires the reference trajectory to have a range equal to the greatest range desired (ref. 5). A little thought will convince the reader that, for ranges which are appreciably shorter than the reference value, severe mismatching of the state variables will occur. That is, the region of valid linearization will have been exceeded, and the linear theory will not provide acceptable guidance.

Unlike time, the total change in velocity which occurs during reentry is independent of the total range traversed. Thus the reference trajectory can be chosen to represent the mean of the range of desired conditions, thus minimizing the mismatching of the state variables. Obviously, the advantage of using an independent variable for which the δx_m of equation (1) are minimized is that less modification to the linear theory is required to make it operate over the range of conditions desired.

At the present time there is no clear indication of the superiority of one set of dependent variables over another. In the present investigation, variables were chosen which would be readily available from an inertial system on board the vehicle, and which did not have obvious disadvantages such as the altitude errors associated with such a system. The state variables chosen were altitude rate, $\dot{h}$, aerodynamic acceleration, A, and range to go, x_{TG}. The control variable, the ratio of the vertical component of lift to drag, L_V/D, was used to control the final range x_f. With these choices, the control equation determined by the linear theory is given by equation (B10) as

$$\left(\frac{L_V}{D}\right)(V) = \left(\frac{L_V}{D}\right)_r(V) + F_{\dot{h}}(V)\delta\dot{h}(V) + F_A(V)\delta A(V) - F_x(V)\delta x_{TG}(V) \qquad (2)$$

Mode of Control

As formulated, the system defined by equation (2) attempts to use minimum control excursion for a maximum length of time: If the information possessed by the system is correct in the sense that the effects of all pertinent quantities have been accounted for, and if the system is in the neighborhood of the nominal trajectory where linearization of the equations is valid, equation (2) will command a control increment just sufficient to drive the range error to zero at $V = V_f$.

Another formulation, used in reference 4, is to command the maximum available control excursion for a minimum amount of time. However, it was found that (when attempting to operate far outside the region wherein linearization is valid) this mode of control tended to cause erroneous trajectory excursions from which it was impossible to recover, and so was not satisfactory.

The mode of control finally used was intermediate to these two extremes; the guidance gains were adjusted through the use of empirically determined weighting factors as will be described subsequently.

Reference Trajectory

The altitude-range characteristics of the reference trajectory chosen for this study are shown in figure 1. Several factors were considered in the choice of this trajectory. It was desired that the range capability of the guidance system be sufficient to return at any time from a lunar mission to a single given earth site, which requires ranges as great as 12,500 statute miles. A 6,000 mile nominal trajectory was chosen since it is approximately in the center of the desired range envelope. A high skip type of trajectory was chosen because both the total heat load incurred and the sensitivity of the final range to state variable errors are lower than for trajectories which have relatively low skip altitudes.

Also shown in figure 1 are the trajectory state variables used in the control equation (2), plotted as a function of the independent variable, velocity. It can be seen in this figure that velocity was double valued during both the initial plunge into the atmosphere and during the resulting skip. Although this characteristic of the independent variable could be accounted for by the appropriate logic, in the interests of simplification the velocity variation was assumed to be monotonic. As a result of this assumption, the guidance system operated with erroneous values of the state variables at both initial velocity and near satellite velocity. Nevertheless, acceptable guidance was obtained with the gain modifications to be described.

Guidance Gains and Empirical Weighting Factors

The linear theory guidance gains associated with the chosen state variables are presented in figure 2. The large value of the gains at the lowest velocity shown is a consequence of the approach of the terminal conditions and the loss of vehicle kinetic energy. The large values which occur near satellite velocity are due to the high skip nature of the reference trajectory. The modification to the gains used in the present analysis are indicated by the dashed lines. The gains at the lowest value of velocity were limited to prevent excessive terminal maneuvering. The other limits shown on the gains associated with altitude rate and aerodynamic acceleration were necessary in order that the velocity variation could be assumed monotonic and, in addition, served to de-emphasize the erroneous state variable values resulting from the same assumption.

It was noted earlier in the report that the purpose of this investigation was to determine the feasibility of guiding to ranges of approximately one-half the earth's circumference, using but a single nominal trajectory. The basic vehicle has the capability, if guided properly, of attaining the ranges desired for a band of reentry angles without violating certain constraints usually applied for the purpose of insuring mission safety. These reentry angle limits determine what will be called the vehicle capability limits. Because of the sizable departures from the nominal trajectory required to realize such a capability, it would not be expected that the linear theory results would be sufficient. The question to be answered then was whether the linear theory gains could be modified to accomplish the desired guidance results and, if so, how complex a modification was required. It was found that the desired guidance results could be obtained through modification of the linear theory gains by means of the weighting factors shown in figure 3. The weighting factors were applied to altitude rate and aerodynamic acceleration, were functions of the final range desired at the time of initial reentry, and assumed the values shown by the appropriate curves depending on whether velocity was greater or less than 25,000 feet per second.

Weighting factors expressed as a function of these two variables, total range and velocity, were the simplest which could be found that were able to extend the capability of the linear theory guidance system to the point where it approached the vehicle capability limits. The weighting factors shown do enable the system to approach this capability, and were determined by the simple process of finding the gradient of the range error in the coordinates of altitude rate and aerodynamic acceleration gains, and varying the gains in the proper manner to decrease the error until the vehicle capability limit was reached, or until a minimum was reached. The combination of weighting factors shown in figure 3 is not unique; other combinations were found which also permitted the guidance capability to approach vehicle capability. Thus no optimum set of weighting factors exists. In an actual application, additional constraints not considered in this investigation would sufficiently define the problem such that an optimum set of factors could be determined. The important result to be noted from this investigation is that virtually full vehicle capability can be utilized by the linear theory guidance system when augmented by these simple means.

The final form of the control equation is given by equation (3).

$$\left(\frac{Lv}{D}\right)(V) = \left(\frac{Lv}{D}\right)_r + K_{\dot{h}}(V,x_f)F_{\dot{h}}(V)\delta\dot{h}(V) + K_A(V,x_f)F_A(V)\delta A(V) - F_x(V)\delta x_{TG}(V) \tag{3}$$

This control equation, utilizing the functions presented in figures 1, 2, and 3, was used to obtain all the results present in the following sections.

GUIDANCE CAPABILITY

Standard Conditions

The trajectory shape, aerodynamic acceleration, and control action are illustrated in figure 4 for two guided trajectories of 6,000 mile range. These two trajectories are for initial flight-path angles closely corresponding to the extremes permitted by the vehicle's capability, namely the uncontrolled skip boundary and the maximum acceleration boundary.

The uncontrolled skip boundary is defined as the shallowest reentry angle for which the vehicle can acquire sufficient aerodynamic force so that the trajectory can be controlled to any desired range. For the vehicle considered in this study the boundary is $\gamma_i = -4.7^{\circ}$. The trajectory presented in figure 4(a) is close to this boundary, and illustrates the negative lift necessary to restrict the range to the desired 6,000 miles. The 0.1° difference in entry angle from the uncontrolled skip boundary was maintained as the shallowest reentry angle for all results presented because of the extremely critical nature of trajectory control as the boundary is approached.

The maximum acceleration boundary is defined as the steepest reentry angle for which the vehicle is capable of preventing the acceleration from exceeding the maximum desired. For the 10g limit chosen for this study, the boundary is $\gamma_i = -7.3^{\circ}$. The reentry angle for the trajectory shown in figure 4(b) is on this boundary, as can be seen from the initial peak of the trace of aerodynamic acceleration. It can also be seen that full positive lift was used over most of the trajectory, indicating that another vehicle capability boundary has been approached, the maximum range boundary.

These three boundaries, which delineate the capability of the vehicle, are shown in figure 5 as a function of the final range. Also shown are symbols which represent guided trajectories calculated to determine the guidance capability. All points within the shaded region of figure 5, as well as for all data to be presented subsequently, represent a terminal range error equal to or less than ±10 miles. It can be seen that within these error limits the guidance system is capable of operating over virtually the entire corridor defined by the vehicle itself.

Off-Design Conditions

An item of significant interest in the evaluation of a guidance system is its ability to handle off-design conditions. Three types of off-design conditions were considered in this study:

(1) Reentry from abort conditions

(2) Changes of vehicle L_V/D

(3) Changes of atmospheric density from 1959 ARDC standard

The abort conditions considered were reentries from circular orbit and reentries at a velocity of 32,000 feet per second. The two reentry conditions shown in figure 6(a) were initiated from a circular orbit at an altitude of 600,000 feet with two different values of retrothrust impulse. The trajectories are shown for each reentry condition; maximum range capability ($L_V/D = 0.4$) of the vehicle, maximum guided range, and minimum guided range determined by the 10g limit. The latter two trajectories define the guidance capability. These results show that the thrust applied in orbit to initiate reentry must be used as the primary range control in this type of abort situation: They also show, however, that the guidance system is capable of utilizing almost the full vehicle capability.

The trajectories bounding the guidance capability for reentries at a velocity of 32,000 feet per second are shown in figure 6(b). At this velocity and at the shallow reentry angle shown, the vehicle has the capability of global range, whereas the guidance system provides only 7,500 miles of that capability. At the steeper angle, the guidance is able to utilize almost full vehicle capability.

The L_V/D variations considered in this investigation were 5 percent above and below the desired value. That is, the actual L_V/D of the vehicle was always 5 percent above or below the value commanded by equation (3). The effect on trajectory shape is shown in figure 7 for reentry at a steep angle. The effects at shallow angles were much less pronounced.

The density change from the 1959 ARDC atmosphere used as the standard atmosphere in this study varied linearly from zero at 100,000 feet to ±50 percent at 400,000 feet altitude. The effects of this deviation on trajectory shape are illustrated in figure 8.

A summary of the effects of these off-design conditions on guidance capability at various ranges is presented in figure 9. Also shown is the information regarding vehicle and guidance capability under standard conditions previously presented in figure 5. The symbols indicate the effects of L_V/D and density variations considered separately. The effects of these off-design conditions on the vehicle capability boundaries were small and are not shown. It can be seen that the L_V/D variations affected the guidance capability relatively little. The density changes, however, caused a sizable loss in guidance capability at long ranges. It is anticipated that inclusion of a component in the control equation sensitive to density changes (the adaptive feature of ref. 6) will make a marked improvement in the guidance capability at long ranges. Another possibility for the improvement of the long range guidance capability in the presence of atmosphere deviations may be the use of a different set of weighting factors.

SUMMARY OF RESULTS

In this study a modified perturbation theory has been applied to the problem of earth reentry guidance. It has been shown that if velocity is used as the independent variable in the control equation and if the linear theory gains are appropriately weighted, then one reference trajectory can be used successfully in spite of large errors in nominal or initial conditions. The use of a single reference trajectory means that the guidance method requires little storage capacity.

It was found that with a single reference trajectory it was possible to obtain guidance for ranges from 1,500 to 12,000 miles over virtually all entry conditions within the vehicle's capability.

For the abort conditions considered in the paper, the guidance system generally was able to make almost full use of the vehicle's range capability.

Errors in vehicle L_V/D considered had little effect on the capability of the guidance scheme.

Density variations considered in this study affected the long range guidance but had little effect on guidance capability for ranges less than 6,000 miles.

APPENDIX A

BASIC EQUATION USED IN LINEAR PERTURBATION GUIDANCE

Developments similar to that in this appendix may be found in the literature (e.g., ref. 7). Consider the set of nonlinear differential equations

$$\dot{x}_m = F_m(x_n, u_p, v) \tag{A1}$$

where $1 \leq n \leq M$

$F = M$ known functions

$x = M$ state variables

$u = P$ external force variables

$v =$ independent variable (such as time, velocity, etc.)

Expanding equation (A1) in a Taylor series about some desired nominal or reference trajectory and retaining only terms to first order gives

$$\delta\dot{x} - \sum_{M} a_{mn}\delta x_n = \sum_{P} b_{mp}\delta u_p \tag{A2}$$

This is a set of M linear differential equations with varying coefficients $a_{mn}(v)$ and $b_{mp}(v)$, the solution of which describes the motion about the reference trajectory, where

$$\delta x_n(v) = x_n(v) - x_{n_r}(v)$$

$$x_{mn}(v) = \left(\frac{\partial F_m}{\partial x_n}\right)_r (v)$$

$$b_{mp}(v) = \left(\frac{\partial F_m}{\partial u_p}\right)_r (v)$$

The set of equations adjoint to equation (A2) is defined by

$$\dot{\lambda}_m + \sum_{M} a_{nm}\lambda_n = 0 \tag{A3}$$

Multiplying equation (A2) by λ_m, equation (A3) by δx_m, summing over M and integrating over the interval v to v_f $(v_1 \leq v \leq v_f)$ gives

$$\sum_M \lambda_m \delta x_m \bigg|_{v_f} = \sum_M \lambda_m \delta x_m \bigg|_v + \int_v^{v_f} \sum_M \sum_P b_{mp} \lambda_m \delta u_p \, dv_1 \tag{A4}$$

This is the basic equation for control about a reference condition, and was called by Bliss (ref. 8) the fundamental formula. Equation (A4) may be particularized by identifying the single sum at $v = v_f$ with the state variable x_q, which it is desired to control $(1 \leq q \leq M)$. Thus, identify

$$\sum_M \lambda_m \delta x_m \bigg|_{v_f} = \delta x_q \bigg|_{v_f} \tag{A5}$$

Then

$$\lambda_m \bigg|_{v_f} = \frac{\partial x_q}{\partial x_m} \bigg|_{v_f} \tag{A6}$$

To indicate the proper partial derivative, the following notation has been introduced in the literature. Equation (A6) is written

$$\lambda_{x_m}^{x_q}(v_f) = \frac{\partial x_q}{\partial x_m} \bigg|_{v_f} \tag{A7}$$

Equation (A7) defines the boundary conditions necessary for the solution $\lambda_{x_m}^{x_q}(v)$ of equation (A3). Equation (A4) may now be written

$$\delta x_q(v_f) = \sum_M \lambda_{x_m}^{x_q}(v) \delta x_m(v) + \int_v^{v_f} \sum_M \sum_P b_{mp} \lambda_{x_m}^{x_q} \delta u_p \, dv_1 \tag{A8}$$

Equation (A8) is the basic equation by means of which an estimate can be made of the first-order change δx_q of the state variable x_q from its reference value at the final condition v_f, due to (a) a change δx_m of any state variable x_m from its reference value at a prior condition v, and (b) a change δu_p of any external force variable u_p from its reference value during the interval v to v_f.

For simplicity, consider u_p to be control variables, and assume the number q of state variables it is desired to control is equal to the number P of control variables. Then, given a desired final value $\delta x_q(v_f)$ and given certain departures $\delta x_m(v)$, there is an infinity of control variable functions which will accomplish the desired final value. In particular, there is a constant value δu_p over the interval $v \leq v_1 \leq v_f$ which will accomplish the desired final value, and, with the notation

$$I_{u_p}^{x_q}(v) = \int_v^{v_f} \sum_M b_{mp} \lambda_{x_m}^{x_q} \, dv_1 \tag{A9}$$

equation (A8) may be written

$$\delta x_q(v_f) = \sum_M \lambda_{x_m}^{x_q}(v) \delta x_m(v) + \sum_P I_{u_p}^{x_q}(v) \delta u_p \tag{A10}$$

Solution of equation (A10) for the control variables u_p then gives

$$u_p(v) = u_{p_r}(v) + \sum_M F_{x_m}^{u_p}(v) \delta x_m(v) \tag{A11}$$

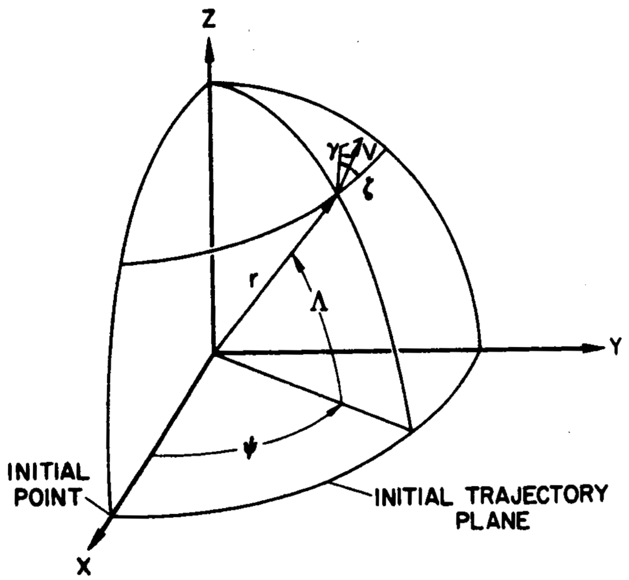
APPENDIX B

DEVELOPMENT OF SPECIFIC CONTROL EQUATION

In this appendix it first will be demonstrated that to first order the two-dimensional results presented in this study are valid for three-dimensional applications. The equations of motion used were

$$\left.\begin{aligned}
\dot{h} &= V \sin \gamma \\
\dot{x} &= r_e\dot{\psi} = \frac{r_e}{r}\,\frac{V \cos \gamma \cos \zeta}{\cos \Lambda} \\
\dot{\gamma} &= \frac{L_V}{mV} + \frac{V}{r}\cos \gamma - \frac{1}{V}\frac{\mu}{r^2}\cos \gamma \\
\dot{V} &= -\frac{D}{m} - \frac{\mu}{r^2}\sin \gamma \\
\dot{\zeta} &= \frac{L_h}{mV}\,\frac{1}{\cos \gamma} - \frac{V}{r}\tan \Lambda \cos \gamma \cos \zeta \\
\dot{y} &= r_e\dot{\Lambda} = \frac{r_e}{r} V \cos \gamma \sin \zeta
\end{aligned}\right\} \qquad \text{(B1)}$$

where the geometry is shown in sketch (a). The assumptions made in the development were:



Sketch (a)

(a) C_D = constant

(b) C_L = constant

(c) Planar reference trajectory

With these assumptions the control variable may be considered to be the vertical component of lift, and the coefficients of the perturbation equation (A2) are found to be, for those which are other than zero,

$$a_{13} = V_r \cos \gamma_r$$

$$a_{14} = \sin \gamma_r$$

$$a_{21} = -(r_e V_r \cos \gamma_r)/r_r^2$$

$$a_{23} = -(r_e V_r \sin \gamma_r)/r_r$$

$$a_{24} = (r_e \cos \gamma_r)/r_r$$

$$a_{31} = -\frac{\beta}{2} \rho_r V_r \left(\frac{C_D S}{m}\right)\left(\frac{L_V}{D}\right)_r - \left(\frac{V_r}{r_r^2} - \frac{2}{V_r}\frac{1}{r_r}\frac{\mu}{r_r^2}\right) \cos \gamma_r$$

$$a_{33} = \left(\frac{1}{V_r}\frac{\mu}{r_r^2} - \frac{V_r}{r_r}\right) \sin \gamma_r$$

$$a_{34} = \frac{\rho_r}{2}\left(\frac{C_D S}{m}\right)\left(\frac{L_V}{D}\right)_r + \left(\frac{1}{r_r} + \frac{1}{V_r^2}\frac{\mu}{r_r^2}\right) \cos \gamma_r$$

$$a_{41} = \frac{\beta}{2} \rho_r V_r^{\,2} \left(\frac{C_D S}{m}\right) + \frac{2}{r_r}\frac{\mu}{r_r^2} \sin \gamma_r$$

$$a_{43} = -\frac{\mu}{r_r^2} \cos \gamma_r$$

$$a_{44} = -\rho_r V_r \left(\frac{C_D S}{m}\right)$$

$$\pm a_{51} = \mp \frac{\beta}{2} \rho_r V_r \left(\frac{C_D S}{m}\right) \frac{1}{\cos \gamma_r} \sqrt{\left(\frac{L}{D}\right)^2 - \left(\frac{L_V}{D}\right)_r^2}$$

$$\pm a_{53} = \pm \frac{\rho_r}{2} V_r \left(\frac{C_D S}{m}\right) \frac{\sin \gamma_r}{\cos^2 \gamma_r} \sqrt{\left(\frac{L}{D}\right)^2 - \left(\frac{L_V}{D}\right)_r^2}$$

$$\pm a_{54} = \pm \frac{\rho_r}{2} \left(\frac{C_D S}{m}\right) \frac{1}{\cos \gamma_r} \sqrt{\left(\frac{L}{D}\right)^2 - \left(\frac{L_V}{D}\right)_r^2}$$

$$a_{56} = -\frac{V_r}{r_r r_e} \cos \gamma_r$$

$$a_{65} = \frac{r_e}{r_r} V_r \cos \gamma_r$$

$$b_{31} = \frac{\rho_r}{2} V_r \left(\frac{C_D S}{m}\right)$$

$$\pm b_{51} = \mp \frac{\rho_r}{2} V_r \left(\frac{C_D S}{m}\right) \frac{1}{\cos \gamma_r} \frac{\left(\frac{L_V}{D}\right)_r}{\sqrt{\left(\frac{L}{D}\right)^2 - \left(\frac{L_V}{D}\right)_r^2}}$$

where the $\pm$ signs indicate the possibility of a left or right orientation of the horizontal component of lift. Then equations (A2) and (A3) become

$$\begin{bmatrix} \delta\dot{h} \\ \delta\dot{x} \\ \delta\dot{\gamma} \\ \delta\dot{V} \\ \delta\dot{\zeta} \\ \delta\dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_{13} & a_{14} & 0 & 0 \\ a_{21} & 0 & a_{23} & a_{24} & 0 & 0 \\ a_{31} & 0 & a_{33} & a_{34} & 0 & 0 \\ a_{41} & 0 & a_{43} & a_{44} & 0 & 0 \\ \pm a_{51} & 0 & \pm a_{53} & \pm a_{54} & 0 & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & 0 \end{bmatrix} \begin{bmatrix} \delta h \\ \delta x \\ \delta\gamma \\ \delta V \\ \delta\zeta \\ \delta y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{31} \\ 0 \\ \pm b_{51} \\ 0 \end{bmatrix} [\delta(L_V/D)] \tag{B2}$$

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \\ \dot{\lambda}_4 \\ \dot{\lambda}_5 \\ \dot{\lambda}_6 \end{bmatrix} = - \begin{bmatrix} 0 & a_{21} & a_{31} & a_{41} & \pm a_{51} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ a_{13} & a_{23} & a_{33} & a_{43} & \pm a_{53} & 0 \\ a_{14} & a_{24} & a_{34} & a_{44} & \pm a_{54} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{65} \\ 0 & 0 & 0 & 0 & a_{56} & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} \tag{B3}$$

and equation (A4) becomes:

$$\begin{aligned} &[\lambda_1\delta h + \lambda_2\delta x + \lambda_3\delta\gamma + \lambda_4\delta V + \lambda_5\delta\zeta + \lambda_6\delta y]_{t_f} \\ &\qquad = [\lambda_1\delta h + \lambda_2\delta x + \lambda_3\delta\gamma + \lambda_4\delta V + \lambda_5\delta\zeta + \lambda_6\delta y]_t \\ &\qquad + \int_t^{t_f} (\lambda_3 b_{31} \pm \lambda_5 b_{51})\delta(L_V/D)dt_1 \end{aligned} \tag{B4}$$

It is desired to control the final downrange, x, and crossrange, y. By the strict identity (A5), the left side of equation (B4) will equal the downrange change at the final value of the independent variable if

$$\left.\begin{aligned} \lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0 \\ \lambda_2 = 1 \end{aligned}\right\} t = t_f \tag{B5}$$

A slightly different formulation holds if the stopping condition is other than the independent variable acquiring some specified value (see, e.g., refs. 7 or 8). In the present type of problem the results are not significantly different. In the notation of equation (A7), equations (B5) are

$$\left.\begin{array}{c}\lambda_{h}^{x} = \lambda_{\gamma}^{x} = \lambda_{V}^{x} = \lambda_{\zeta}^{x} = \lambda_{y}^{x} = 0 \\ \lambda_{x}^{x} = 1\end{array}\right\} t = t_f \tag{B6}$$

Solving equations (B3) using the boundary conditions (B6) gives the values for all t. By inspection

$$\left.\begin{array}{c}\lambda_{\zeta}^{x} = \lambda_{y}^{x} = 0 \\ \lambda_{x}^{x} = 1\end{array}\right\} \text{for all } t \tag{B7}$$

That is, to first order, there is no effect of heading angle, ζ, or crossrange, y, on the downrange, x. This result is true only for a planar trajectory, a restriction approximately fulfilled as a result of the nature of the vehicle considered in this paper; the strictly two-dimensional results presented should then remain valid if extended to a full three-dimensional investigation.

Equation (B4) now may be written

$$\delta x(t_f) = [\lambda_{h}^{x}\delta h + \delta x + \lambda_{\gamma}^{x}\delta\gamma + \lambda_{V}^{x}\delta V]_t + \int_t^{t_f} \lambda_{\gamma}^{x} b_{31}\delta(L_V/D)dt_1 \tag{B8}$$

Transformation to any independent variable and combination of state variables is a simple matter. As noted in the text, velocity was chosen as the independent variable, and altitude rate, $\dot{h}$, aerodynamic acceleration, A, and range, x, were the state variables chosen. With these variables, equation (B8) becomes

$$\delta x_f = \delta x(V_f) = [\lambda_{\dot{h}}^{x}\delta\dot{h} + \delta x + \lambda_{A}^{x}\delta A]_V + \int_V^{V_f} \lambda_{\gamma}^{x} b_{31}\delta(L_V/D)dV_1 \tag{B9}$$

and equation (A11) becomes

$$\left(\frac{L_V}{D}\right) = \left(\frac{L_V}{D}\right)_r + F_{\dot{h}}\delta\dot{h} + F_A\delta A + F_x(\delta x - \delta x_f) = \left(\frac{L_V}{D}\right)_r + F_{\dot{h}}\delta\dot{h} + F_A\delta A - F_x\delta x_{TG} \tag{B10}$$

where

$$I^x_{L_V/D} = \int_V^{V_f} \lambda^x_\gamma b_{31}\, dV_1$$

$$F_x = \frac{1}{I^x_{L_V/D}}$$

$$F_A = \frac{\lambda^x_A}{I^x_{L_V/D}}$$

$$F_{\dot{h}} = \frac{\lambda^x_{\dot{h}}}{I^x_{L_V/D}}$$

and the superscript has been left off the F functions because of the single control variable.

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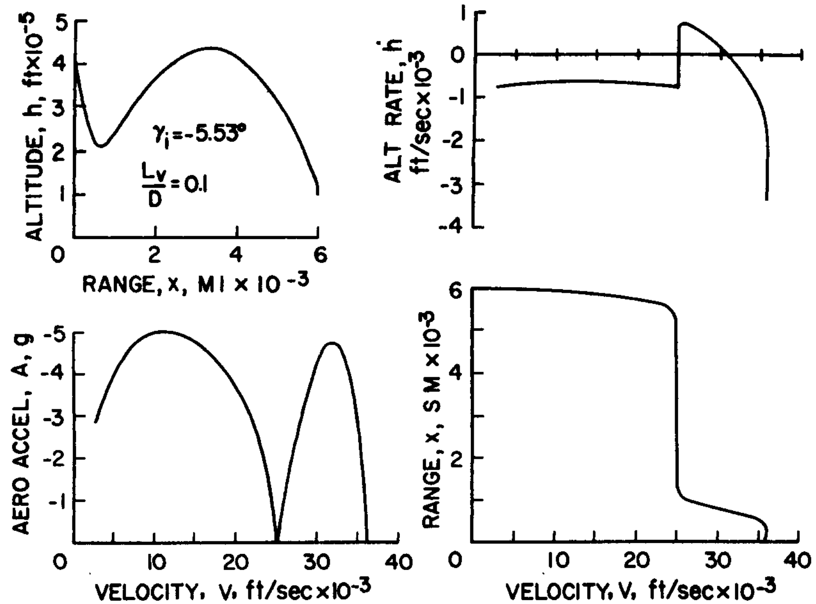


Figure 1. - Reference trajectory state variables.

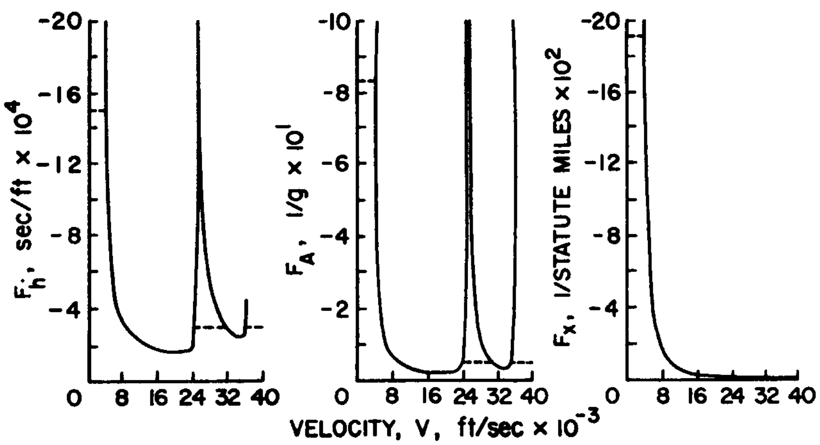


Figure 2. - Linear theory guidance gains.

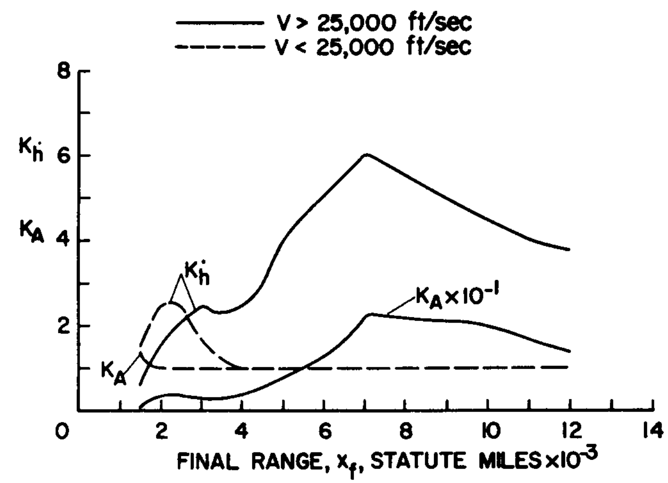
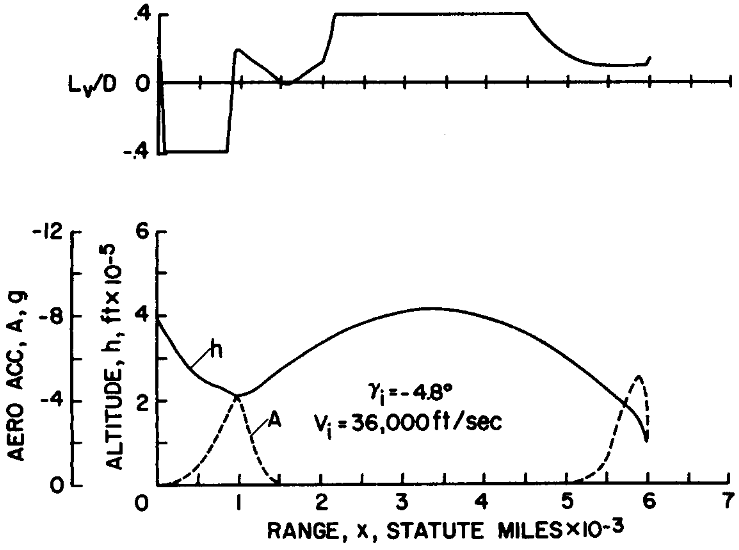
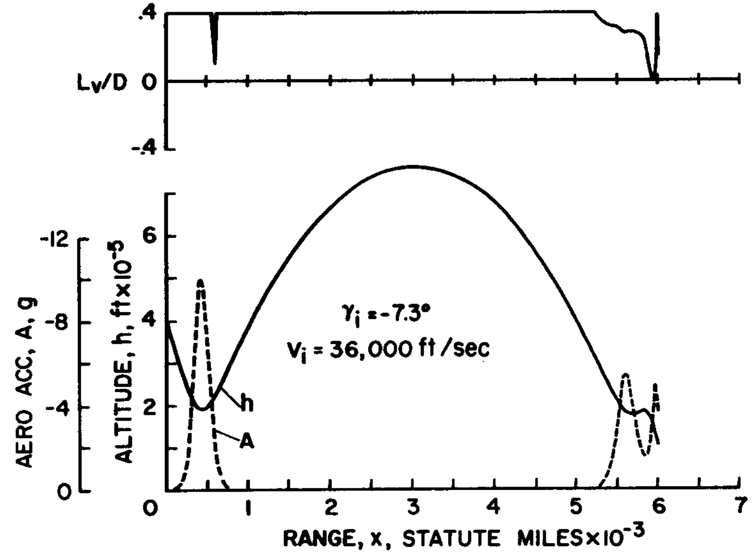


Figure 3. - Empirical weighting factors.



(a) Shallow entry.

Figure 4. - Typical guided trajectories.



(b) Steep entry.

Figure 4. - Concluded.

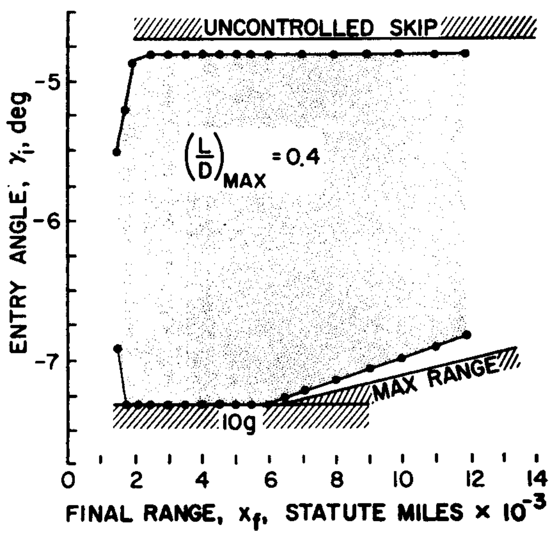
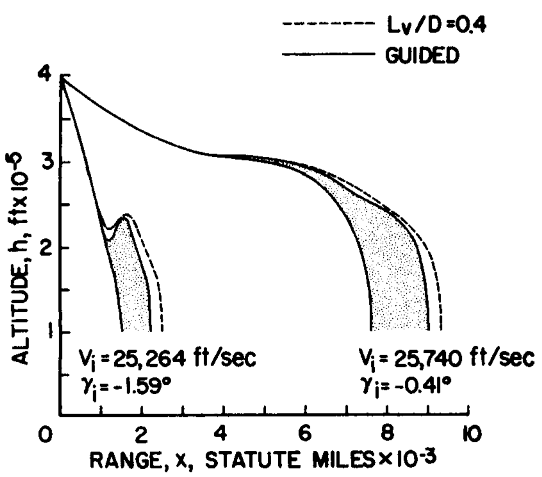
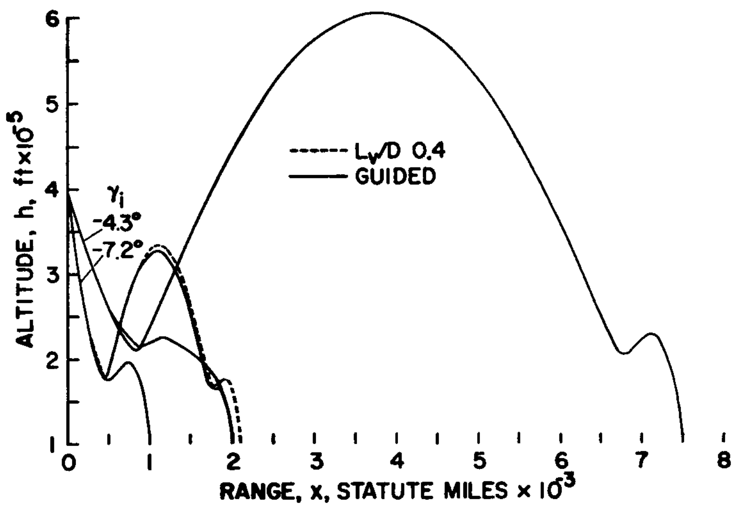


Figure 5. - Guidance capability.



(a) Reentry from circular orbit.

Figure 6. - Abort conditions.



(b) Reentry at 32,000 feet per second.

Figure 6. - Concluded.

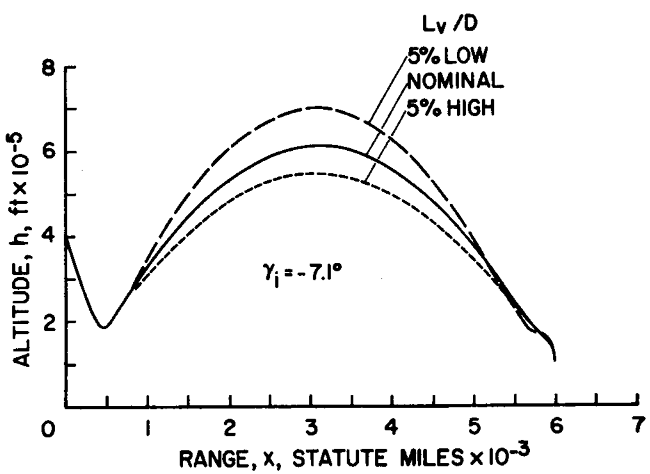


Figure 7. - Effect of L_V/D variation on guided trajectory.

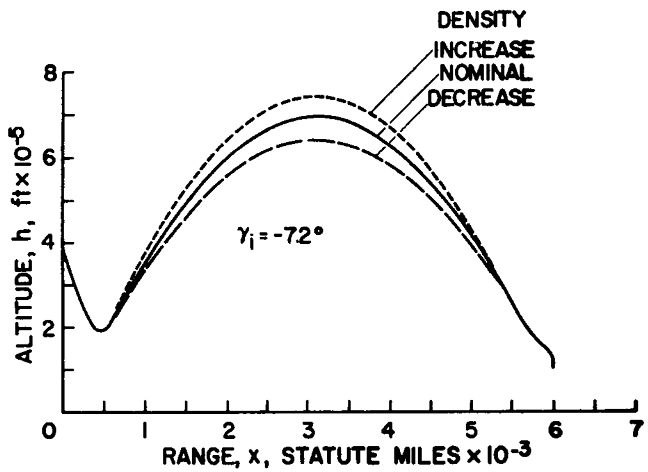


Figure 8. - Effect of density variation on guided trajectory.

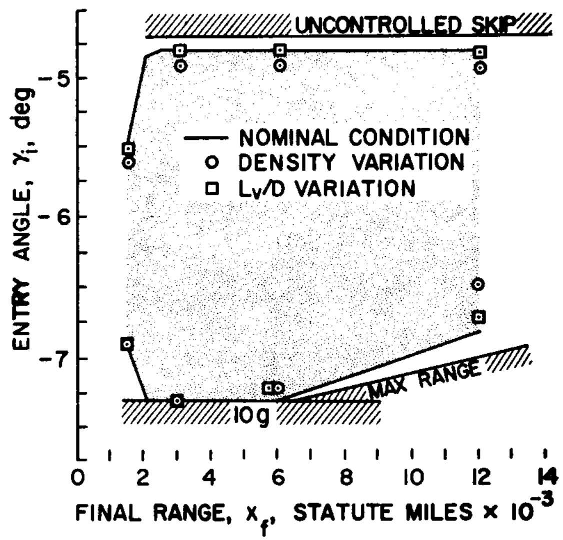


Figure 9. - Effect of off-design conditions on the guidance capability.